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Mandibular shape and skeletal divergency

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SUMMARY Pre-treatment lateral cephalograms of 41 skeletal Class I girls aged 11 to 15 were divided according to MP–SN angle: lower than 28 degrees (hypodivergent, 10 girls), between 31 and 34 degrees (normodivergent, 18 girls), or larger than 37 degrees (hyperdivergent, 13 girls). The mandibular outlines were traced and digitized, and differences in shape were quantified using the elliptic Fourier series. Size differences were measured from the areas enclosed by the mandibular outlines. Shape differences were assessed by calculating a morphological distance (MD) between the size-independent mean mathematical reconstructions of the mandibular outlines of the three divergency classes.

Mandibular shape was different in the three classes: large variations were found in hyperdivergent girls versus normodivergent girls (MD = 4.61), while smaller differences were observed in hypodivergent girls (MD versus normodivergent 2.91). Mean size-independent mandibular shapes were superimposed on an axis passing through the centres of gravity of the condyle and of the chin. Normodivergent and hyperdivergent mandibles differed mostly at gonion, the coronoid process, sigmoid notch, alveolar process, posterior border of the ramus, and along the mandibular plane. A significant size effect was also found, with smaller mandibles in the hyperdivergent girls.

Introduction

The analysis of the radiographic projection of craniofacial structures is an essential step in the diagnosis and treatment planning of dentofacial deformities. Since the first application of radiography to dentistry, several cephalometric analyses have been proposed for the study of size and shape of hard and soft tissue structures, as well as for their reciprocal arrangement (Merow and Broadbent, 1990). Most of the measurements involved the maxilla and the mandible; that is, the jaw bones that articulate along with the dentition. Likewise, these bones can be affected by non-surgical treatment.

The lateral radiographic projection of the mandible is almost free from superimpositions of other bones. Its outline could thus be traced from conventional cephalograms with sufficient precision. A quantitative analysis of mandibular shape could provide valuable information concerning the whole arrangement of the facial and cranial skeleton (Kerr *et al.*, 1994). Indeed, modifications of the size and shape of single bones can influence the position, as well as the form, of other hard and soft tissue structures, in order to maintain a harmonious and balanced whole (Enlow *et al.*, 1969; Ferrario *et al.*, 1997).

Apart from the intrinsic limitations of two-dimensional projections of three-dimensional structures (Fine, 1994), linear measurements sufficiently evaluate the dimensions of craniofacial components. Unfortunately, most of the analyses are chiefly concerned with craniofacial shape, and with the reciprocal arrangement of structures, and a considerable number of angles and linear ratios have been devised in the attempt of shape quantification. For instance, mandibular shape is quantitatively described by the gonial angle, the inclination of the body (mandibular plane), ramus and condyle relative to the sella-nasion line or to Frankfort horizontal (Merow and Broadbent, 1990; Halazonetis *et al.*, 1991; Karlsen, 1995). Maxillo-mandibular relationships are estimated with angular measurements relative to the anterior cranial base (point A–nasion–point B angle, nasion–point A–pogonion angle), to Frankfort horizontal (facial angle), or with linear ratios (Wits index; Merow and Broadbent, 1990; Halazonetis *et al.*, 1991; Hurmerinta *et al.*, 1997). In particular, the position of the lower mandibular border (mandibular plane) relative to the sella-nasion line has also been associated with the resultant direction of facial growth (Karlsen, 1995), and mandibular shape (as assessed by the gonial angle) has recently been reported to be significantly correlated with the spatial orientation of the jaw muscles (van Spronsen *et al.*, 1997), and with bite force (Ingervall and Minder, 1997). None of these measurements deals with the intrinsic mandibular shape, indeed, they could be modified also by alterations in the cranial base and anterior facial structures (Merow and Broadbent, 1990).

More correct analyses of the shape of biological structures can be performed by Fourier series. This method mathematically describes the outline of objects, and can quantitatively analyse their global shape characteristics, independently from their size, spatial orientation, or relationship to reference planes. In particular, size could be a confounding factor in the analysis of shape changes, because modifications in size are often of greater magnitude than the corresponding modifications in shape (Lestrel *et al.*, 1977; Lestrel and Kerr, 1993). Both classic Fourier series and elliptic Fourier analysis have already been successfully applied to the quantitative study of biological forms in several fields: dentistry, osteology, neurology, haematology (for review see Ferrario *et al.*, 1995).

In particular, mandibular form provided by lateral cephalograms has already been analysed by elliptic Fourier analysis, a method best suited for closed contours (Lestrel, 1989; Lestrel and Kerr, 1993; Lowe *et al.*, 1994; Ferrario *et al.*, 1996). Previous investigations, however, have never assessed if skeletal divergency and mandibular shape are associated, i.e. if individuals with different mandibular plane to sella-nasion line angles have differently shaped mandibles.

The purpose of the current investigation was to determine whether or not skeletal divergency can influence the intrinsic (i.e. size and orientation independent) morphological characteristics of the radiographic image (lateral projection) of the mandible in children of the same age, gender, and race, but of different divergency classes. Fourier analysis, in this investigation, was used to separate the size and shape effects, and to supply a global quantitative evaluation of the shape discrepancy, that may lead the clinician toward a more accurate diagnosis and the best available treatment.

Subjects and methods

Sample

One-hundred-and-forty pretreatment lateral cephalograms of adolescent orthodontic female patients aged between 11 and 15 years were used. Age was rounded up to the nearest 6 months, i.e. all patients between 10 years, 6 months and 1 day, and 15 years, 5 months and 29 days, were considered. This particular age range was selected to match the adolescent growth spurt for European and North American children (12 years for girls; Tanner, 1978). The patients were all white Caucasians (Italian) and had dental malocclusions correctable by orthodontic treatment only, as judged by an expert orthodontist. No surgical patients were evaluated. All the subjects were admitted to a private practice during a 3-year period. All the cephalograms were obtained in the standing posture, using a Siemens Ortho P10 cephalometer (Siemens AG, Germany), 18×24-cm films and a final enlargement of 10 per cent. Details of the radiographic technique have been reported previously (Ferrario *et al.*, 1994a). The Harvard–Forsyth cephalometric analysis (Moorrees *et al.*, 1976) was performed on all of the patients. Cephalograms were then analysed according to the following criteria:

1. Skeletal class: only skeletal Class I patients (corrected ANB angle between 0 and 4

degrees) were investigated; the corrected ANB angle was computed as: original ANB angle + 0.5 × (81.5 degrees – SNA angle) + 0.25 × (32 degrees – SN–GoMe angle; Miralles *et al.*, 1991);
2. Skeletal divergency, as defined by the mandibular plane to sella-nasion line angle (MP–SN): lower than 28 degrees (hypodivergent), between 31 and 34 degrees (normodivergent), or larger than 37 degrees (hyperdivergent);
3. A well defined mandibular outline.

Forty-one girls matched the criteria. They were divided into three divergency classes as detailed in Table 1. The three classes did not differ significantly in age (one-way analysis of variance, 2;38 degrees of freedom, $F = 1.081$, $P > 0.05$). Borderline values for the MP–SN angle (i.e. between 28 and 30 degrees, and between 35 and 37 degrees) were not considered, the analysis being mostly concerned with differences between well-defined and not overlapping classes.

A combination of reference measurements was used to set the MP–SN angle for this study: the Harvard–Forsyth analysis (threshold between normo- and hyper-divergent patients: MP–SN angle 37 degrees; Moorrees *et al.*, 1976), the Steiner analysis (reference value for MP–SN angle 32 degrees; Steiner, 1953), a study by Isaacson *et al.* (1971) who defined as hypodivergent all patients with MP–SN angle less than 26 degrees and hyperdivergent all patients with MP–SN angle greater than 38 degrees, the investigation by Lowe *et al.* (1994) where all patients with MP–SN angle equal to or larger than 37 degrees were classified as long-face or hyperdivergent, a study by Karlsen (1995) who defined hypodivergency with MP–SN angles of 26 degrees or less, and hyperdivergency with MP–SN angles of 35 degrees or more, and the investigation by Hurmerinta *et al.* (1997) where the threshold between low MP–SN angle (hypodivergent) and mean angle patients was set at 28 degrees, and that between mean angle and high-angle (hyperdivergent) patients at 38 degrees.

The mandibular outlines of all girls were carefully traced from the radiographs by a single operator to ensure consistency. The technique consisted of tracing the outline of the mandible beginning near articulare with the posterior aspect of the rami, the gonial angles, the inferior borders, the symphysis, the bone overlying the roots of the lower incisors, the alveolar crest, the coronoid process and mandibular notch closest to the film (the most inferior on the cephalogram), and the mandibular condyle. With the exception of the coronoid process, mandibular notch, and condyle, the average was taken between any double lines of left and right structures.

Table 1 Number of girls in each divergency class, mean age, and mean area enclosed in the mandibular outline.

Girls	Hypo	Normo	Hyper
Number	10	18	13
Age* (years)	12.55	12.12	12.84
SD	1.24	1.09	1.63
Area** (mm^2)	2573.27	2494.57	2313.02
SD	282.92	260.90	170.70

*No significant difference between the three classes ($P > 0.05$, one-way analysis of variance).
**Significant difference between the three classes ($P < 0.05$, one-way analysis of variance).

Fourier analysis

The subsequent analysis was performed by a computer, programmed by one of the authors (VFF). The outline of each mandible was digitized by an automatic image analyser (IBAS, Kontron, Munich, Germany; Ferrario *et al.*, 1994b, 1996), and the *x*- and *y*- co-ordinates of approximately 150 points for each outline were obtained. The co-ordinates of these points were encoded with a numeric code (or chain code) as described by Kuhl and Giardina (1982), and Ferrario *et al.* (1996), which provided a description of the mandibular outline, and allowed the calculation of the relevant elliptic Fourier coefficients, supplying a mathematical and harmonic description of the outline itself (Kuhl and Giardina, 1982; Ferrario *et al.*, 1994b, 1996).

This numeric code approximates a continuous contour by a sequence of piecewise linear fits that consist of eight standardized line segments. The code of a contour is then the chain V of length K:

$$V = a_1 a_2 a_3 \dots a_k,$$

where each link a_i is an integer between 0 and 7 orientated in the direction $(\pi/4)a_i$ (as measured counter-clockwise from the x-axis of an x–y co-ordinate system), and of length 1 or 2 depending on whether a_i is even or odd, respectively.

The elliptic Fourier series expansions for the x- and y-projections of the encoded contour are defined as

$$x(t) = A_0 + \sum_{n=1}^{20} (a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T})$$

$$y(t) = C_0 + \sum_{n=1}^{20} (c_n \cos \frac{2n\pi t}{T} + d_n \sin \frac{2n\pi t}{T})$$

where A_0 and C_0 are the co-ordinates of the harmonic centroid; a_n, b_n, c_n, d_n are the four coefficients of the nth elliptic harmonic; and $0 < t \leq T$, where T is the basic period of the chain code (Kuhl and Giardina, 1982).

The elliptic Fourier series were normalized with respect to the rotation, translation, and size of the contour. This allowed the study of mandibular shape independently from its size and spatial orientation. Standardization for rotation was performed by aligning the semi-major axis of the first harmonic on the abscissa; for translation, by ignoring the A_0 and C_0 terms; and for size, by dividing each coefficient by the magnitude of the first semi-major axis (Kuhl and Giardina, 1982). Fourier series were truncated at the 20th harmonic, because the higher degree coefficients and relevant amplitudes were negligible.

Before size standardization, the area enclosed in each mandibular outline was automatically computed.

Evaluation of the effect of divergency

Shape. Mean mandibular shapes within divergency class were computed. A 'morphological distance' (MD) (i.e. a measurement of differences in shape) between mean mandibles was computed using the relevant Fourier coefficients (Kuhl and Giardina, 1982; Ferrario *et al.*, 1994b, 1996). MD measures the Euclidean distance in a 20-dimensional space (the first 20 harmonics) between two plots characterized by their Fourier coefficients, which are used like the Cartesian co-ordinates in standard metric measurements. MD equals 0 when the profiles are identical, and when they have been sampled with the same lattice coarseness and orientation (Kuhl and Giardina, 1982; Ferrario *et al.*, 1994b, 1996). For easier reading, all morphological distances were multiplied by 100.

In the present protocol, the lattice was automatically plotted by the image analyser with a standardized coarseness (i.e. number of x, y lines for linear unit), and mandibular orientation was carefully checked. Therefore, MD is likely to measure only real differences between pairs of mandibular plots. MD was named 'morphological' because it measures the differences in shape, independently from size, spatial orientation, and relationship to reference planes (Ferrario *et al.*, 1994b, 1996). MD quantifies the difference between two outlines (in the present investigation, between mean mandibular outlines in the three divergency classes) by a single number which is comprehensive of all the morphological characteristics of the analysed structures (i.e. it is no longer necessary to look at separate—and possibly discordant—angular and linear measurements such as the gonial angle, the inclination of the symphysis, the corpus, and ramus length).

The normalized Fourier coefficients were used to plot the mean mandibles for each divergency class. Mean plots were superimposed on the centroid of the first harmonic, and on an axis passing through the centres of gravity of the condyle and of the chin (approximately between pogonion and menton), and the locations of the main shape differences were assessed.

Size. Size differences were measured from the areas enclosed by the mandibular outlines and, taking into account the magnification of the radiographic film, were transformed into square millimetres. A one-way analysis of variance was performed between the three divergency classes.

Error of method

Provided the films are of good quality, the only significant source of error of this investigation was in mandibular tracing. Ten randomly chosen cephalograms were retraced by the same operator with a 1-month interval, redigitized, and the relevant Fourier coefficients calculated. The MD between each outline and the mean normodivergent mandible was computed, and the difference between each pair of distances calculated (Ferrario *et al.*, 1994b). No systematic errors were found (mean distance 0.039, SD 0.20, $P > 0.10$, Student's *t*-test for paired samples). The repeated tracings were compared with a common reference. However, the choice of the reference itself does not influence the results, it being only the differences between the MD which were assessed (Ferrario *et al.*, 1994b).

Correspondingly, the areas enclosed between the mandibular outlines were computed, and differences between the first and second tracing compared. No systematic errors were found (mean difference 0.015, SD 0.0261, $P > 0.10$). Other negligible errors could derive from the mathematical procedure, in particular from the approximation algorithms.

Results

In all children the elliptic Fourier series gave an excellent superimposition between the original mandibular plot and its mathematical reconstruction.

On average, hyperdivergent girls had the smallest mandibles, while hypodivergent girls had the largest ones (Table 1). Variability was limited and similar in all three classes (coefficients of variation, the percentage ratio of standard deviation to mean, between 7 and 11 per cent). Size differences were significant between the three divergency classes (one-way analysis of variance, $F = 2.593$, 2;38 degrees of freedom, $P < 0.05$). *Post-hoc* Student's *t*-tests found significant differences ($P < 0.05$) between the areas of the mandibular outlines of hyper- and hypo-divergent girls, and between hyper- and normo-divergent girls.

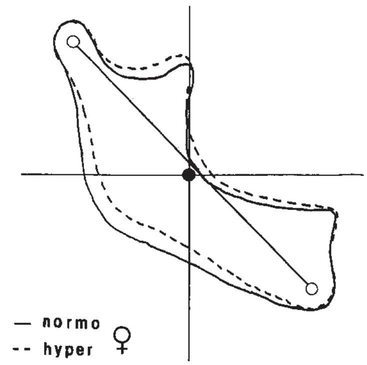

Figure 1 Size-standardized mean mandibular outline in normo- (continuous line) and hyper-divergent (interrupted line) girls. The outlines are superimposed on the centroid of the first harmonic, and on an axis passing through the centres of gravity of the condyle and of the chin. MD is 4.61.

The mean mandibular outlines of the three divergency classes had different size-standardized shapes. Large differences were found in hyperdivergent girls: the MD in normo- versus hyper-divergent girls was 4.61, while smaller differences were observed in hypodivergent girls (MD versus normodivergent 2.91). As expected, the morphological distance between hypo- and hyper-divergent mandibles was the largest (MD = 5.75). To assess the areas that differed most between the mandibles in the three divergency classes, mean outlines were superimposed on an axis passing through the centres of gravity of the condyle and of the chin (approximately between pogonion and menton). This axis almost corresponded to the semi-major axis of the first harmonic of the elliptic Fourier series expansion. The size-independent shapes of the normo- and hyper-divergent mandibles differed mostly at gonion, coronoid process, sigmoid notch, alveolar process, posterior border of the ramus, and along the mandibular plane (Figure 1). When compared with the mandibles of normodivergent girls, the mandibles of hyperdivergent girls had a steeper mandibular plane with a somewhat larger gonial angle, a more anterior ramus, a higher alveolar process, and a more concave sigmoid notch.

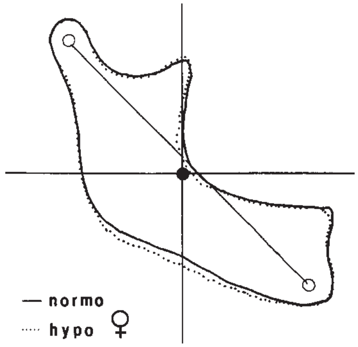

Figure 2 Size-standardized mean mandibular outline in normo- (continuous line) and hypo-divergent (dotted line) girls. The outlines are superimposed on the centroid of the first harmonic, and on an axis passing through the centres of gravity of the condyle and of the chin. MD is 2.91.

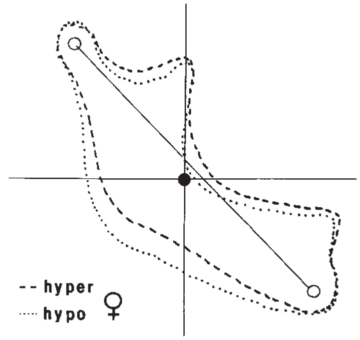

Figure 3 Size-standardized mean mandibular outline in hypo- (dotted line) and hyper-divergent (interrupted line) girls. The outlines are superimposed on the centroid of the first harmonic, and on an axis passing through the centres of gravity of the condyle and of the chin. MD is 5.75.

Conversely, the differences between normo- and hypo-divergent mandibles were less manifest, though interesting the same parts that differed most between normo- and hyperdivergent mandibles: mandibular plane, alveolar process, sigmoid notch (Figure 2). Differences were most noticeable when the size-standardized mean mandibles of the two extreme classes were superimposed (Figure 3).

Discussion

Hyperdivergent girls had a significantly smaller lateral plane radiographic projection of the mandible than normo- and hypo-divergent girls of the same age, skeletal class, and race. This size difference was coupled with a significant shape difference: hyperdivergent mandibles appeared to develop somewhat more anteriorly and superiorly than normo- and hypo-divergent mandibles.

In the sample analysed in the present study, therefore, different skeletal vertical relationships (the reciprocal positions of mandible and cranial base) were associated with different mandibular sizes and shapes. Indeed, the analysis did not involve a formal correlation between divergency and mandibular form: only three well-defined, non-overlapping classes were analysed, and their mean mandibular outlines evaluated. Moreover, the study was cross-sectional and no investigations concerning the growth behaviour of the analysed girls were performed.

Previous investigations on skeletal divergency

The literature reports several studies that analyse mandibular 'form' and skeletal divergency, but, with the exception of a study by Lowe *et al.* (1994), no mathematical analyses of mandibular size and shape were performed. Several conventional cephalometric measurements were made, and mandibular form (without any size-standardization) was described by sets of angles and distances. Karlsen (1995) investigated craniofacial growth in a group of 30 boys from 6 to 15 years of age. In the children with a low MP–SN angle (26 degrees or less), the length of the mandibular corpus increased more than in children with a high MP–SN angle (35 degrees or more). Moreover, between 6 and 12 years, the gonial angle decreased more in the low angle group than in the high angle group, while the mental process increased the most in the high angle group. Differences were also found in the inclination of the symphysis, and in the length of the mental process. The results of the present study cannot be directly compared with those obtained by Karlsen (1995) because of several differences in the sample characteristics (boys versus girls, longitudinal versus cross-sectional sampling, age interval), and, obviously, in the methods. Nevertheless, his findings of a different behaviour of the gonial angle in the two divergency groups, and of a larger increase of the mandibular corpus in hypodivergent boys, are in agreement with the current results: larger mandibular area (size) and gonial angle (shape) in the hypodivergent girls.

Siriwat and Jarabak (1985), who classified their 500 children aged 8–12 according to the facial height ratio (S–Go to N–Me ratio), found that children with a hyperdivergent face had a short mandibular ramus and a large gonial angle. The average facial pattern of hypodivergent children was the opposite. Again, these results obtained with conventional cephalometric assessments are in accord with the current mean Fourier plots, but the effects of mandibular dimension and shape cannot be separated.

Analysis of mandibular shape by Fourier series

Cephalometric analyses often deal with individuals with great size differences and, to better appreciate the shape of structures as well as in their reciprocal arrangement, some control for size variability should be performed. For instance, Moorrees' mesh diagram analysis (Moorrees *et al.*, 1976) applies an internal linear reference to standardize for different craniofacial dimensions.

Unfortunately, the radiographic projection of the mandible is a complex shape, which cannot be reduced into Euclidean geometry and correctly considered by the conventional metric measurements (Fine, 1994; Lowe *et al.,* 1994). Moreover, it is difficult to select the best and most informative measurements. These limitations resulted in the use of both elliptic Fourier analysis and classic Fourier series in the quantification of the differences in shape in cephalometric tracings, as recently reviewed by Ferrario *et al.* (1996).

Fourier series allow a correct quantitative analysis of shape and of shape differences without imposing any geometric constraint to the mandible and a correct size-standardization. Moreover, Fourier analysis is a true shape analysis, because normalized Fourier coefficients are position-, size-, and orientation-invariant.

Lestrel and Kerr (1993) applied the elliptic Fourier series to quantify the effect of functional therapy on the maxillary and mandibular shape identified by several standardized landmarks. Fourier series were used to describe the shape and the shape changes of the outline of these bones in the lateral cephalometric radiographs of 14 patients with a Class III malocclusion. Lowe *et al.* (1994) used the elliptic Fourier series to quantify the shape of the maxilla, mandible, and cranial base in a large group of patients. Again, the outlines of these bones were reconstructed from a series of standardized landmarks. The consistency between the conventional cephalometric classification of anterior-posterior and vertical relationships, and clustering according to Fourier descriptors was evaluated. Indeed, the two criteria were only marginally in agreement, with an overall error rate of 33 per cent. Ferrario *et al.* (1996) quantified the differences in shape between the mandibular tracings of the Bolton standards from 1 to 18 years of age.

There are some differences between the present investigation and previous studies performed using the elliptic Fourier series, both in the Fourier reconstruction (digitization, bone outline, superimposition) and in the quantification of differences between groups. In the current investigation, as well as in the one by Ferrario *et al.* (1996), mandibular outline was traced from the cephalograms, and automatically digitized. However, both Lestrel and Kerr (1993) and Lowe *et al.* (1994) reconstructed their mandibular forms from closely spaced cephalometric landmarks. For instance, in the study by Lestrel and Kerr (1993), the mandible was reconstructed from 78 landmarks, while in the current study approximately 150 points for each mandible were used. On this basis, the present mandibular reconstruction should be more accurate. Mandibular reconstruction was truncated at 20 harmonics, because the higher degree coefficients were negligible. A similar truncation was made in the previous analyses performed on the radiographic projection of the same bone (Lestrel and Kerr, 1993; Lowe *et al.*, 1994; Ferrario *et al.,* 1996).

In the current study, the differences between the mean mandibular shapes of the three divergency classes were computed from the coefficients of the Fourier series, and a morphological distance was computed (Kuhl and Giardina, 1982; Ferrario *et al.*, 1994, 1996). A similar quantification was not performed by Lestrel and Kerr (1993) and Lowe *et al.* (1994). Its main advantage is that it gives a single number which locates unequivocally the shape differences and allows an easier rating of structures.

Conclusions

The present investigation quantified the shape differences between the mandibular cephalometric tracings independently from the size differences. The observed size and shape differences between skeletal divergency classes were compatible with previous findings performed with conventional cephalometrics, but their effect was separately assessed.

Mandibular form (as assessed by the gonial angle) and mandibular position relative to the cranial base have been associated with the resultant direction of facial growth, the spatial orientation of the jaw muscles, and the bite force (Karlsen, 1995; Ingervall and Minder, 1997; van Spronsen *et al.*, 1997), and all these factors could be implied in the different mandibular sizes and shapes found in the present study. Unfortunately, it is not simple to assess if the different mandibular morphology is the cause or the consequence of a different muscular orientation and force, and it is even more difficult to quantify the effect of each single factor. Investigations performed on patients undergoing a surgical correction of mandibular prognathism seem to indicate an alteration of the mechanical advantage of the masticatory muscles, but its direction is difficult to predict in the individual patient (Mavreas and Melsen, 1997).

The Fourier series can be used to compare mandibular tracings from patients where size differences partially mask the shape differences; for instance, in longitudinal studies or when individuals of both genders are involved. Moreover, the calculation of the morphological distance between the mandibular outlines supplies a global quantitative evaluation of the shape discrepancy, and may lead the clinician toward a more accurate diagnosis and the best available treatment (Ferrario *et al.*, 1991). The mean mandibular outlines of hypo-, normo-, and hyper-divergent girls computed in the present study may provide a first reference for the comparison of new outlines.

The investigation could also be extended to quantify the separate size and shape effects of either orthodontic or combined surgical orthodontic treatment on selected malocclusions and craniofacial malformations, in a wide age range of patients.

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